Clock model interpolation and symmetry breaking in O(2) models[1]

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QuLAT Collaboration

[1]On arXiv soon!

Outline

1. The Model
   - Monte Carlo results
2. The $\gamma \to \infty$ Limit
   - Monte Carlo results at small volume
   - Phase transitions for $q \in \mathbb{Z}$
   - TRG results at large volume
   - Phase transitions for $q \notin \mathbb{Z}$
3. Conclusion

The Model

- We consider a classical spin model in 2D with energy function

$$S = -\beta \sum_{x,\mu} \cos\left(\varphi_{x+\hat{\mu}} - \varphi_x\right) - \gamma \sum_x \cos(q\varphi_x)$$

- The spins reside on lattice sites and take values $\varphi \in [0, 2\pi)$.
  1. When $\gamma = 0$, this is the classic XY model, with a BKT transition
  2. When $\gamma \to \infty$ and $q \in \mathbb{Z}$, this is the classic q-state clock model with discrete $\mathbb{Z}_q$ symmetry in which the spins are forced into the values

  $$\varphi \in \left\{0, \frac{2\pi}{q}, \frac{4\pi}{q}, \cdots, \frac{2\pi(q-1)}{q}\right\}$$

     For $q = 2, 3, 4$ there is a second order phase transition and for $q \geq 5$ there are two phase transitions
  3. When $\gamma \to \infty$ but $q \notin \mathbb{Z}$, we get a particular continuation of the q-state clock model to non-integer q

Some Monte Carlo Results

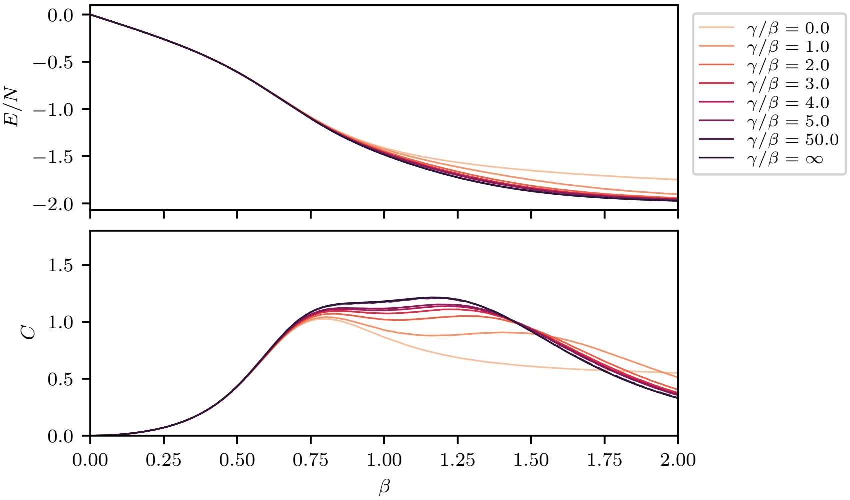


Figure: Results from a Metropolis algorithm on a 4×4 lattice with $q = 5.0$.

The $\gamma \to \infty$ Limit

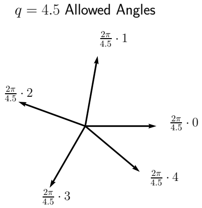
- For the $\gamma \to \infty$ limit, we replace the energy function with

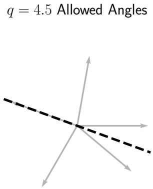
$$S = -\beta \sum_{x,\mu} \cos\left(\varphi_{x+\hat{\mu}} - \varphi_x\right),$$

where the spins are

$$\varphi \in \left\{0, \frac{2\pi}{q}, \frac{4\pi}{q}, \cdots, \frac{2\pi\lfloor q \rfloor}{q}\right\}$$

- For $q \in \mathbb{Z}$, we have the ordinary q-state clock model
- For $q \notin \mathbb{Z}$, we get something new





Some Monte Carlo Results

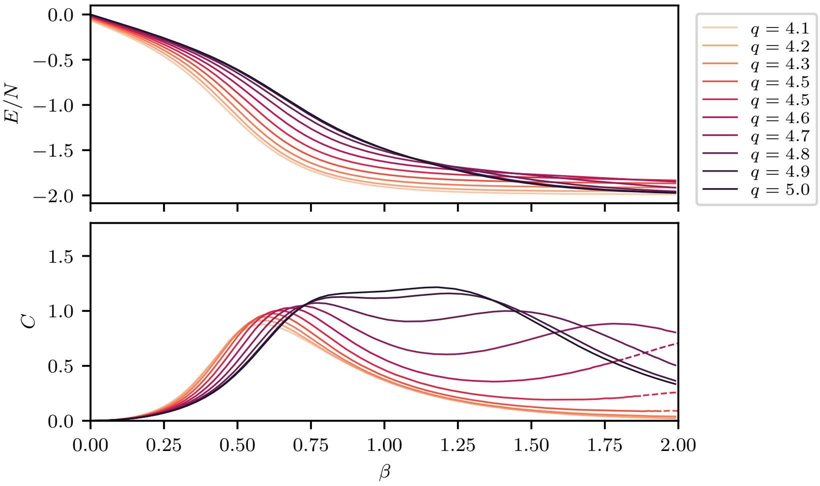


Figure: Results from a heatbath algorithm on a 4×4 lattice.

Phase transitions for $q \in \mathbb{Z}$

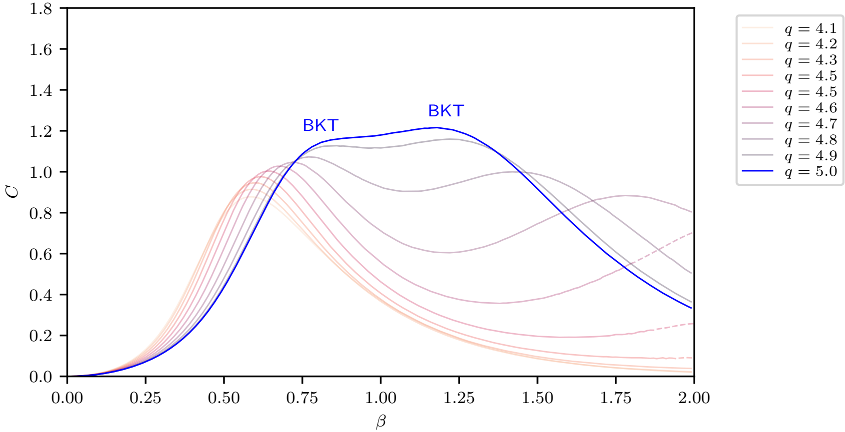


Figure: For integer $q \geq 5$, both peaks in the specific heat are associated with BKT transitions (see Li et. al. in Phys. Rev. E **101**, 060105(R) (2020)).

Phase transitions for $q \notin \mathbb{Z}$

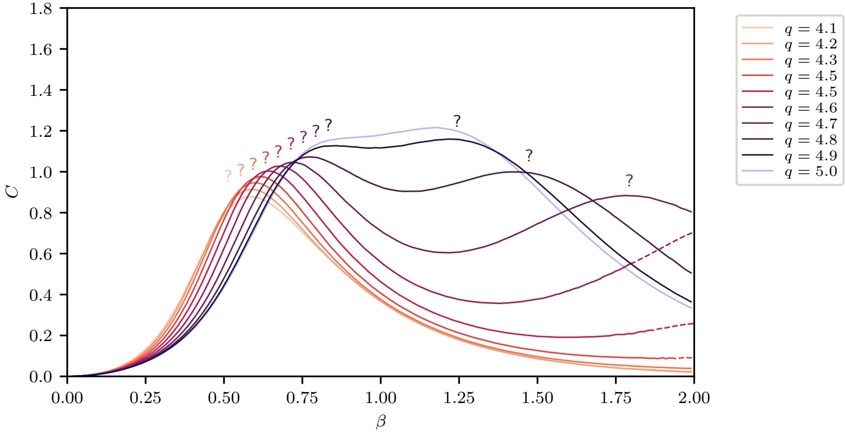
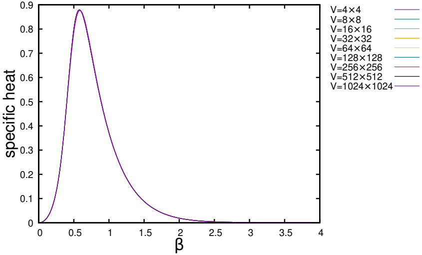


Figure: For non-integer q, are these also BKT transitions or are they something else?

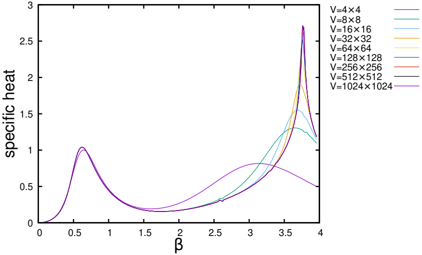
TRG

- In the Monte Carlo approach, we use a Markov chain importance-sampling algorithm to generate equilibrium configurations
  - Unfortunately, in this model for non-integer q and large β, the configuration space splits into two sectors and the Markov chain tends to get stuck in one or the other
  - Monte Carlo has difficulty sampling this model appropriately at $\beta > 1$ for $q \notin \mathbb{Z}$
  - Integrated autocorrelation time explodes, and we have to perform billions of heatbath sweeps already on a 4×4 lattice
  - Studying this model on larger lattices with Monte Carlo is challenging
- Tensor renormalization group (TRG) approach can be used instead
  - We validate TRG against Monte Carlo in the regime accessible to Monte Carlo
  - Then we use TRG to explore lattice sizes and β-values beyond the reach of Monte Carlo

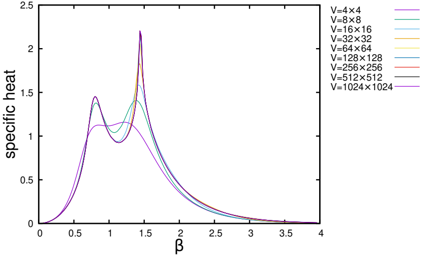
Some TRG Results



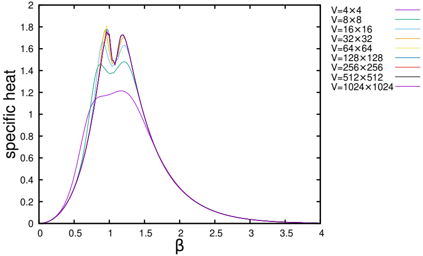
(a) $q = 4.1$



(b) $q = 4.5$



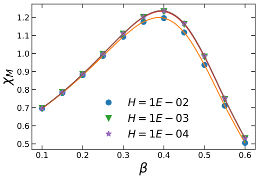
(c) $q = 4.9$



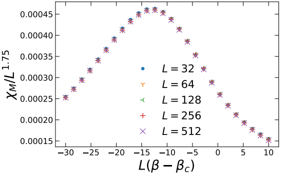
(d) $q = 5.0$

Phase transitions for $q \notin \mathbb{Z}$

- First peak in specific heat:
  - Magnetic susceptibility converges to a finite constant as $H \to 0$
  - Magnetic susceptibility converges to a finite constant as $V \to \infty$
  - $\implies$ Not a phase transition
- Second peak in specific heat:
  - Looking at the magnetization vs. magnetic susceptibility gives result consistent with $\delta = 15$
  - Finite size scaling of magnetic susceptibility gives result consistent with $\gamma = 7/4$
  - $\implies$ Ising phase transition

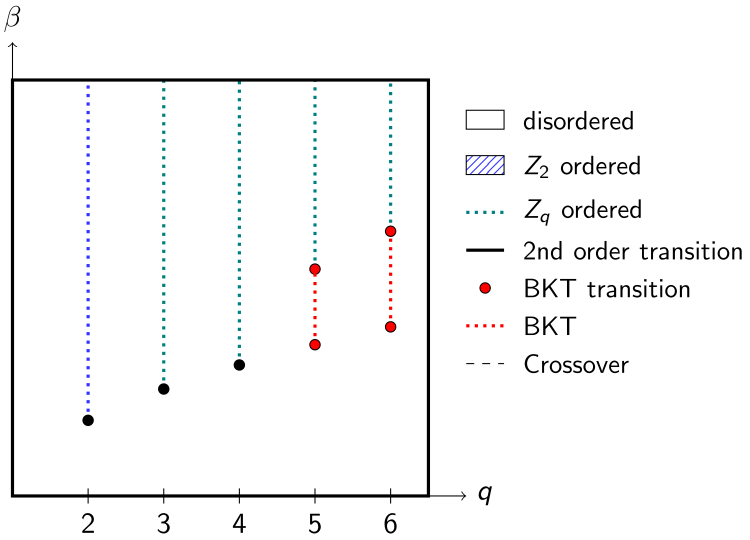


Example from TRG with $q = 4.3$ showing χ_M going to a constant as external field is taken to zero.

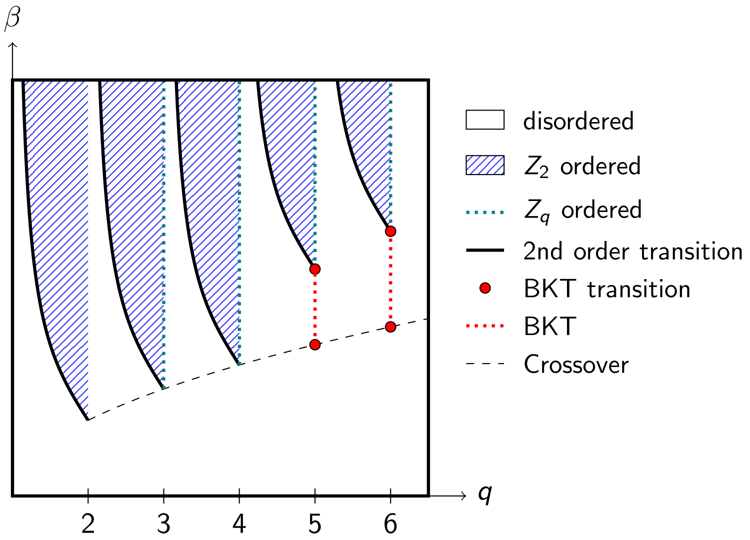


Example from TRG with $q = 4.3$ showing that volume dependence of χ_M is consistent with $\gamma = 7/4 = 1.75$.

Phase diagram for $\gamma = \infty$ and $q \in \mathbb{Z}$ (i.e. clock model)



Phase diagram for $\gamma = \infty$ and $q \in \mathbb{R}$



Conclusion

1. We looked at an extended O(2) model with parameters β, γ, and q

$$S = -\beta \sum_{x,\mu} \cos\left(\varphi_{x+\hat{\mu}} - \varphi_x\right) - \gamma \sum_x \cos(q\varphi_x)$$

2. We learned enough to establish the $\gamma = \infty$ slice of the 3D phase diagram
   - When $q \in \mathbb{Z}$, we recover the classic q-state clock model which has a single second-order phase transition for $q = 2, 3, 4$ and two BKT transitions for $q \geq 5$
   - When $q \notin \mathbb{Z}$, we get a crossover and a second-order phase transition
3. This model may be a good candidate for analog quantum simulation as a small step toward the ultimate goal of simulating QCD on a quantum computer
4. Perhaps on Rydberg arrays. See (Keesling et. al. Nature **568**, 207 (2019)) and talk by Yannick Meurice (M31.00008)

Any questions?

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Some Monte Carlo Results

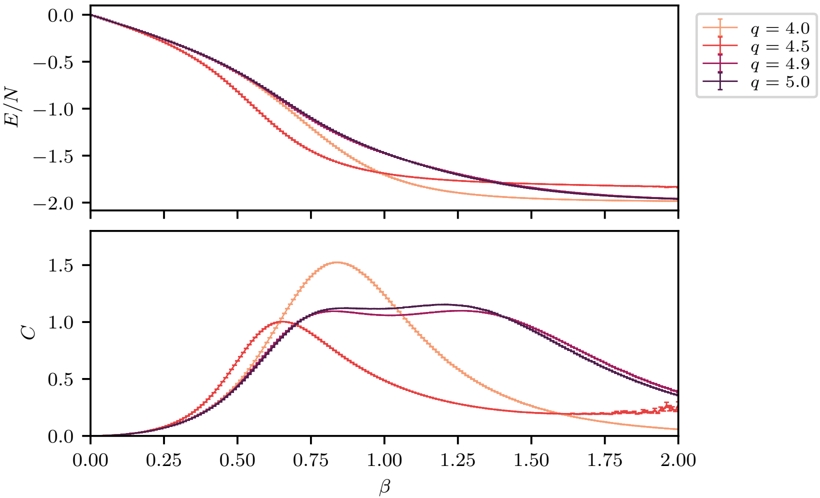


Figure: Results from a Metropolis algorithm on a 4×4 lattice with $\gamma/\beta = 5.0$.

Some Monte Carlo results for $\gamma \to \infty$

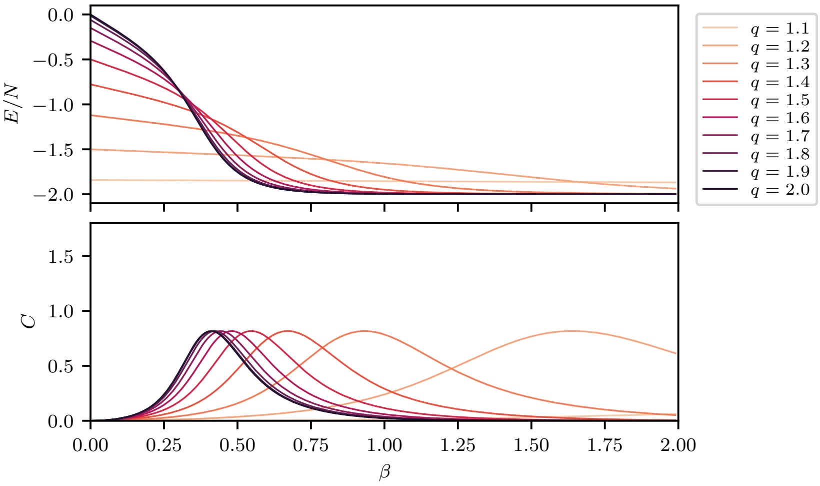
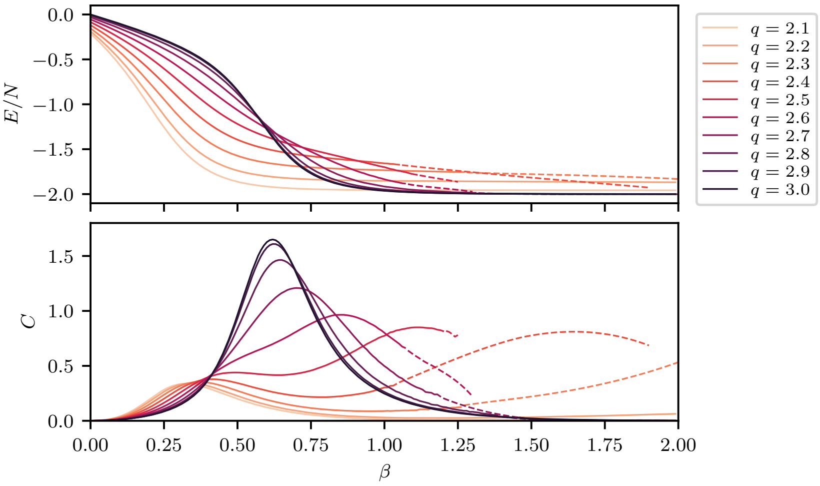


Figure: Results from a heatbath algorithm on a 4×4 lattice.

Some Monte Carlo results for $\gamma \to \infty$



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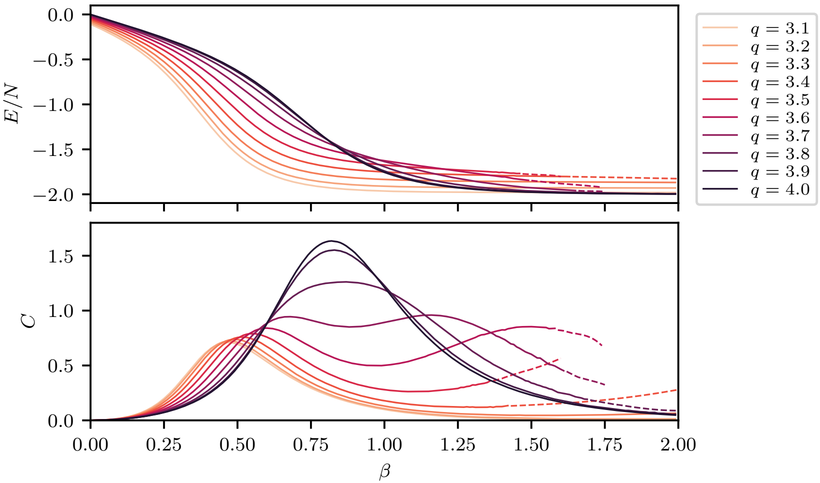

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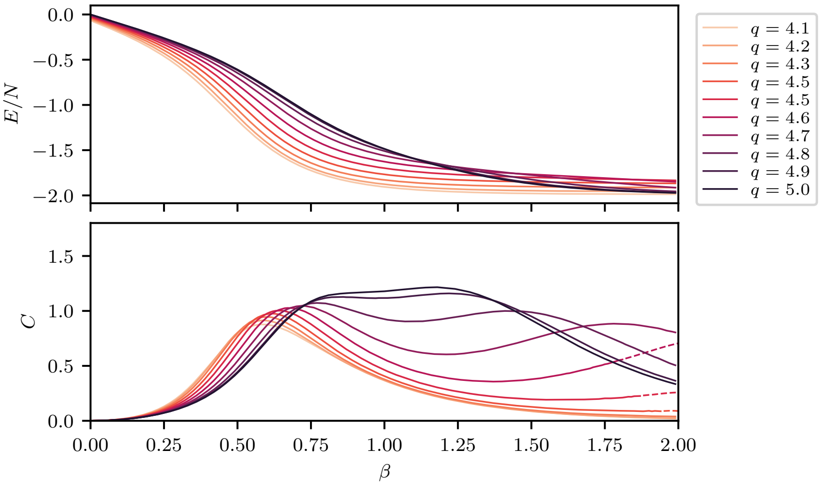

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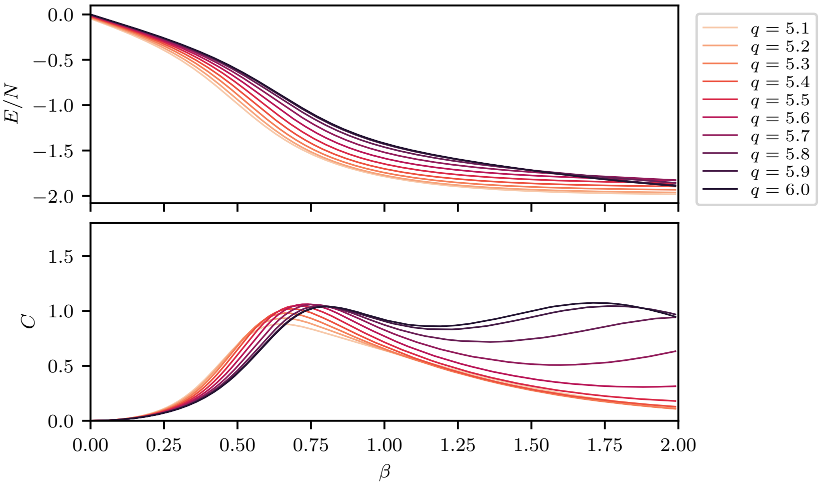
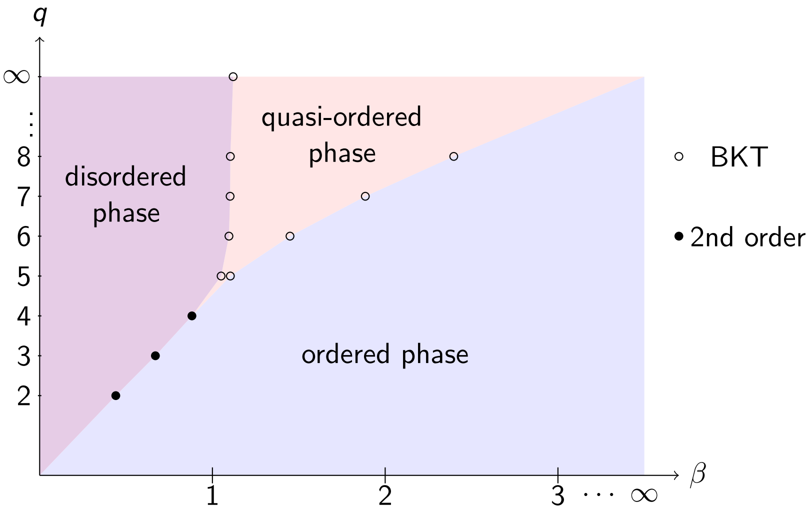
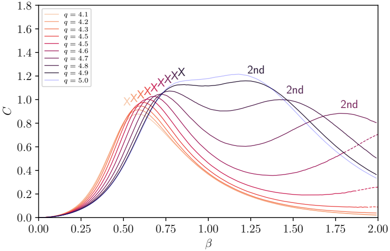


Figure: Results from a heatbath algorithm on a 4×4 lattice.

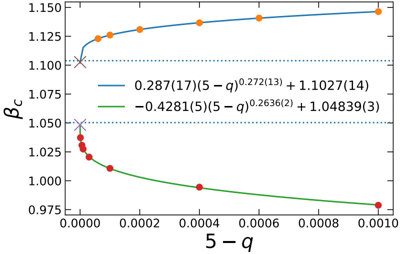
Phase diagram for integer q



Phase transitions for $q \notin \mathbb{Z}$



For non-integer q, the small-β peak in the specific heat is associated with a crossover, and the large-β peak is associated with a 2nd order transition.



Power law extrapolation of the peaks in the magnetic susceptibility as $q \to 5$ for zero external field.